Name: \_\_SOLUTIONS\_\_\_\_\_\_\_\_

****

Full Test (Calc Free + Calc Assumed)

Total Time: 60 minutes

Total Marks: 50 marks

Student Result \_\_\_\_\_\_\_\_/ 50

**MATHEMATICS METHODS Unit 3**

**TEST 1 -2023: Further differentiation, integration and applications.**

**Calculator Free Section**

Time: 31 minutes

Total Marks: \_\_\_\_\_\_ / 27 marks

Resources allowed: SCSA Formula Sheet

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks**. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 1** | **[2, 2, 2 = 6 marks]** |

Determine for each of the following. Do not simplify your answer.

a)

✓correctly differentiates at least two terms

+✓ correctly differentiates all three terms

b)

✓uses quotient rule

+✓ correctly differentiates

c)

✓chain rule

+✓ correctly differentiates

|  |  |
| --- | --- |
| **Question 2** | **[3 marks]** |

Find the value of for

✓uses quotient rule for 1st derivative

✓correct 2nd derivative (chain rule)

✓correct value for

|  |  |
| --- | --- |
| **Question 3** | **[4, 2 = 6 marks]** |

Consider the function, .

a) Determine the coordinates of all the stationary points for and use the second derivative to determine their nature.

✓correct derivative

✓solves

function is concave up at

has one stationary point, a local minimum at

✓shows use of the second derivative to justify concavity/nature of T.P.

✓correctly states local minimum at

b) Find the coordinates of the global minimum and maximum points of over the

interval .

for all in the interval

function is concave up for

is also a global minimum over the interval.

Testing the bounds

global minimum at and global maximum at

✓correct statement ✓determines

|  |  |
| --- | --- |
| **Question 4** | **[2, 2 = 4 marks]** |

Determine each of the following indefinite integrals. Express you answers with positive indices where appropriate.

a)

✓correct numerator (function)

✓correct denominator (coefficient) and

b)

✓correct coefficient and

✓correct reciprocal function

or

|  |  |
| --- | --- |
| **Question 5** | **[1, 2, 2 = 5 marks]** |

Consider the function shown below.

<EFOFEX>
id:fxd{b685f7a8-0338-4fea-99e1-475995989bfa}

FXData:
</EFOFEX>

Evaluate the following integrals.

a)

✓determines correct value (signed area)

b)

✓applies integral property

✓determines correct value (signed area)

c)

✓applies integral properties

✓determines correct value (signed area)

|  |  |
| --- | --- |
| **Question 6** | **[1, 2 = 3 marks]** |

Determine

a)

✓uses the Fundamental Theorem of Calculus

b)

✓applies definite integral property

✓indicates use of the Fundamental Theorem of Calculus

Name: \_\_SOLUTIONS\_\_\_\_\_\_\_\_

****

**MATHEMATICS METHODS Unit 3**

**TEST 1 -2023: Further differentiation, integration and applications.**

**Calculator Assumed Section**

Time: 29 minutes

Total Marks: \_\_\_\_\_\_ / 23 marks

Resources allowed:

SCSA Formula Sheet

Up to three Calculators and

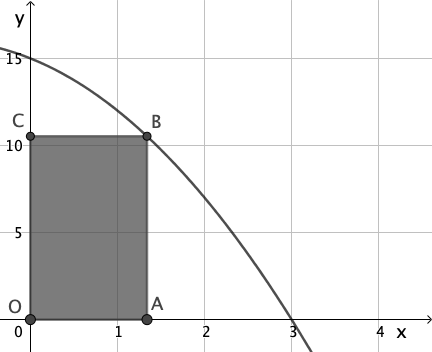
One A4 sheet, both sides of notes

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. **For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 7** | **[1, 2, 4, 2, 1 = 10 marks]** |

A rectangle OABC is such that O is always at the origin, A lies on the -axis, C lies on the -axis and B lies in the first quadrant on the curve .



1 unit on each axis is 1 cm.

a) Find the area of the rectangle when .

✓correct

b) Show that the area of rectangle OABC is given by , where is the -coordinate of corners A and B

✓expands and collect like terms

✓area in terms of only to show what is required

c) Use calculus (first and second derivatives) to determine the maximum area of the rectangle.

\*

or can dismiss ✓solves , CAS

\* ✓\*obtains and derivatives

curve is concave down at local max

✓tests second derivative for concavity/nature

✓correct maximum

d) i) Use the increments formula to find the approximate change in area of the rectangle when increases from 2 to 2.1 cm.

✓correct values used in increments formula

✓correct approximate change in area

ii) Interpret this answer in the context of this question.

The area decreases by approximately as increases from to .

✓correct statement regarding a decreasing area

|  |  |
| --- | --- |
| **Question 8** | **[1, 2, 3, 1 = 7 marks]** |

The rate of flow of water from a reservoir is given by for , where is measured in litres and is the number of minutes since the sluice gates have be further opened. The sluice gates are initially slightly open and are fully open after 24 minutes.

a) Find the rate of flow (in litres per minute) when and .

L/minute

L/minute

✓correct rate of flow for both times

b) Sketch the graph of against for .

<EFOFEX>
id:fxd{20507730-b5a4-460d-94b4-5a0bc2e26113}

FXData:
</EFOFEX>

✓graph bounds are and

✓graph passes through and has approximately the correct shape.

c) Determine the area under the curve of between and .

✓shows required integral

✓correctly integrates: equiv. to

✓correct value for area

d) What does this area represent?

The area (answer to part c)) represents the total volume of water (in L) being released from the reservoir during in the 24 minutes in question.

✓indicates total volume of water

|  |  |
| --- | --- |
| **Question 9** | **[2, 4 = 6 marks]** |

A small body with an initial displacement and velocity of 8 m and respectively, travels in a straight line with acceleration at time of , .

a) Determine an expression for the velocity.

✓integrates acceleration

✓determines value for

b) Calculate the displacement of the body when it has a velocity of .

✓integrates velocity

✓determines value for

Solve for when using CAS

solve ✓solves for time when velocity

m (right of the origin) ✓determines required displacement

**End of Test**